The Role of Confidence for Disputes

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Abstract

In this model, agents with differing views decide which views to tolerate. The connections between these agents depend on their socialization efforts. Any remaining agents are in dispute. Benefits stem from disputes, which are contests between players, and increase in an agent's strength and confidence. An agent's strength is the number and weight of their connections, and their confidence depends on the number of connections who are in dispute with their opponent. The equilibrium network either consists of isolated echo chambers or opponents have mutual connections. Overall dispute intensity decreases in how much confidence agents derive through their connections if society consists of echo chambers and decreases otherwise. Encouraging socialization reduces dispute intensity when society is close to forming echo chambers.

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1 Introduction

The rise of social media in the past decades has contributed substantially to the increasing polarization in society. Polarization divides individuals across ideological lines and holds the potential for social conflict (Simmel, 1955; Coser, 1956). A large body of literature documents structurally larger polarization online (Halberstam and Knight, 2016) and a polarizing effect of social media (Lelkes, Sood, and Iyengar, 2017; Allcott, Braghieri, Eichmeyer, and Gentzkow, 2020; Melnikov, 2022).

In principle, social media shapes polarization through two opposing channels. On the one hand, social media exposes users to more diverse views and content. This pushes towards a consensus. On the other hand, exposure to conflicting views and ideologies challenges the identities of users and results in disputes between them. In a sense, users protect their views and ideologies against what they perceive as an attack by others. Disputes are thus non-constructive confrontations and push towards polarization, since their hostile nature drives social media users further apart.

Importantly, how confident individuals are in their views depends on their interactions. In particular, a user's connections, who are in dispute with the same individual as herself, reassure her of her views and increase her confidence. In a sense, connections create an echo chamber effect, thereby altering interactions between users and potentially intensifying disputes on social media.

This paper connects confidence to disputes and thus polarization. I study a heterogeneous society, where agents hold intrinsic views on societal issues, their identities. Views can represent to which extent an agent believes vaccination should be mandatory, whether environmental preservation justifies higher public spending, or compose of views on multiple topics. A connection between two individuals requires *tolerance* for the other's views. Tolerance is costly to the agents, and more so for views that are very different from the own. Moreover, agents choose how much time to spend on social media, their *socialization effort*. An agent's connections become stronger if she chooses a higher socialization effort, i.e., spends more time on the social media platform. Connections strengthen an agent in disputes with others and increase her confidence. Connections thus allow agents to derive greater benefits from disputes, however, establishing them requires effort and tolerance.

I model disputes as a contest between two players. The players in dispute are *opponents*. Like in the canonical Tullock (1980) contest, benefits are increasing in the own strength and decreasing in the opponent's strength. An agent's strength is the sum of her weighted links.[1]

[1] König, Rohner, Thoenig, and Zilibotti (2017) define the strengths of militias in conflict in a similar way. Agents become stronger in their own fighting effort as well as the fighting effort of their allies.

An agent's strength is a sign of acceptance of her views in society, since she can only establish connections to those, who tolerate her views.[2] For simplicity, agents are in dispute whenever at least one of them does not tolerate the views of the other.[3] I augment the contest success function by a confidence channel. An agent grows confident to "win" a dispute when more of her connections are in dispute with her opponent as well. An agent's connections reassure her of her views, thereby creating an echo chamber effect and distorting perceptions.[4] The value of a connection depends on the entire network. In this sense, confidence pushes towards echo chambers, since agents want to coordinate on having mutual opponents. Agents trade off the value of a connection, i.e., how much the connection increases her strength and confidence, against the tolerance cost it entails. Since confidence distorts perceptions, both players in a dispute can simultaneously believe to "win".

The equilibrium network either consists of echo chambers, or players in dispute have common connections, i.e., the network is an *overlapping society*. Echo chambers are cliques, where every member of the clique is connected to everyone in the same clique and in dispute with everyone outside the clique. Echo chambers form when the relative benefits from confidence are sufficiently high. Players are willing to tolerate even distant types who are in many disputes with opponents of themselves. Those connections boost their confidence in many instances. The benefits from confidence then outweigh the additional burden of tolerance and echo chambers arise. Otherwise, the network is an overlapping society. Players establish connections to more similar types and reduce their burden of tolerance. The additional burden of tolerance outweighs the potential benefits from confidence.

Interestingly, the intrinsic network structure determines the effect of changes to the economy. In particular, I focus on *dispute intensity*. Dispute intensity comprises of the number of disputes and the strengths of the players involved. It is thus a measure of polarization. If the network intrinsically consists of echo chambers, an increase in how much confidence agents can derive through their connections decreases dispute intensity. Higher confidence through connections pushes players to coordinating on many mutual opponents. Thereby, they enjoy the greater benefits from confidence. At least one echo chamber grows large and there are fewer disputes in society. Dispute intensity decreases on the extensive margin. Since there are fewer disputes, players overall exert lower socialization efforts and are of lower strengths. Dispute intensity decreases on the intensive margin as well. In an overlapping society, higher confidence pushes towards echo chambers. Players coordinate on having more mutual oppo-

[2]This notion is different from the notion of popularity, since popularity usually captures incoming links.

[3]In Section 5, I allow for endogenous dispute initiation.

[4]The model nests several commonly accepted behavioral fallacies.These include the availability heuristic Tversky and Kahneman (1973) or an overly optimistic evaluation of connections and oneself Tesser and Campbell (1982); Tesser (1988).

nents. The number of disputes increases and agents exert overall higher socialization efforts. Dispute intensity increases on the extensive and intensive margin.

Another critical component for polarization is how costly socialization is. In this context, easier access to a social media platform reduces the hurdles for socialization, i.e., the socialization cost. Melnikov (2022) documents larger political polarization in areas with access to 3G internet, i.e., easier access to social media, compared to areas without 3G internet. Improving the access to social media is detrimental for society unless society is on the verge of consisting of echo chambers, i.e., for intermediate socialization costs. Then, encouraging socialization pushes towards more overlaps in the neighborhoods of players in dispute, since the relative gains from confidence decrease. Dispute intensity decreases on the extensive margin. Note, as agents socialize more, dispute intensity always increases on the intensive margin. However, if the network is close to echo chambers, the crowing out of disputes, the extensive margin, dominates. For low, respectively high, socialization costs all agents are relatively similar in their strengths. It is thus beneficial to form echo chambers, since the relative benefits from confidence increase. Encouraging socialization aggravates this effect and dispute intensity increases on the extensive and intensive margin.

There are several other important takeaways from the model. First, extremists' behavior goes in the opposite direction to overall effects on society. If extremists are in more disputes, moderates are in fewer, thereby outweighing the polarizing effect of extremists. How easily agents can tolerate differing views determines their tolerance choices. Interestingly, introducing *stubborn* or *flexible* extremists pushes towards echo chambers. This is because either extremists do not want to interact with moderates, or moderates do not want to interact with extremists.

The model is robust to several extensions and can be adapted in several meaningful ways. One can study alternative notions of strength, endogenous dispute initiation, and can feed the static model in a learning model with discrete time. Moreover, a two-dimensional version of the model can predict military alliances based on geography.

This paper relates to the literature on polarization. Esteban and Ray (1994) are the first to establish polarization as a measurable economic outcome and have thus moved the phenomenon to the center of scholars' attention.[5] Other papers study the emergence of polarization through preferences.

The closest to this paper is Genicot (2022), who demonstrates the role of heterogeneity for compromise. In my model, agents cannot shirk their identities. Instead, agents choose to tolerate others with views that differ from their own. However, tolerating different views causes disutility to agents. In many instances, views are fixed in the short run. Examples are

[5] Huremović and Ozkes (2022) extend this framework to networks.

an individual's assessment to which extent potentially lower infection rates justify mandatory vaccination, or to which degree environmental preservation justifies higher public spending. Those are typically fixed during a wave of infections or a political cycle. This paper focuses on such instances. Moreover, the benefits in my model arise from the whole network rather then each individual connection. Thus, incentives to coordinate on mutual opponents arise.

Another strand of literature studies polarization in contexts where payoffs depend on the network and other endogenous actions. Baccara and Yariv (2013, 2016) study polarization when agents join groups and contribute to public projects. Here, confidence may push agents to form echo chambers. In Allmis and Merlino (2023), heterogeneous players form links to access others' contributions to two types of information. Encouraging interaction can have adverse effects on polarization and welfare by crowding out information provision and thus free riding opportunities. Here, encouraging socialization can increase the number of disputes in society and how intense the individual disputes are. A decrease in socialization costs pushes towards cliques, and thus more disputes, when it reduces the differences in agents' strengths.

I model dispute as a contest and borrow various tools from the economics of conflict literature. A contest success function determines the payoffs of the opponents in a dispute (Tullock, 1967, 1980; Hirshleifer, 1989). In König et al. (2017), benefits from conflict depends on own, allies' and enemies' fighting efforts, i.e., the network. Like in their paper, socialization effort ("fighting effort") crowds out the socialization efforts of connections ("allies"), however, encourages socialization of opponents ("enemies"). Hiller (2017) studies a model of alliance formation, where players make friends to extract payoffs from weaker enemies. Here, interactions also depend on identities. Echo chambers emerge only for sufficiently high relative benefits from confidence.

Players have identity-based payoffs and can modify their identity at a cost as in Akerlof and Kranton (2000). The role of identity in my paper is twofold. First, it determines tolerance and thus which players interact. In this sense, there is homophily (McPherson, Smith-Lovin, and Cook, 2001), i.e., the tendency to interact more with similar others, through preferences. Other papers model homophily as biased meetings (Currarini, Jackson, and Pin, 2009; Currarini, Matheson, and Vega-Redondo, 2016). Second, exposure to conflicting views poses a challenge to an agent's identity. Disputes are attempts to restore one's identity. Indeed, this is a common phenomenon on real world social media platforms (Conover et al., 2011, 2012).

I augment the contest success function with a confidence channel. Agents grow more confident in their views, their identity, when their connections have similar opponents as themselves. Arguably, players interact more frequently with their connections and might thus overweigh their support (Tversky and Kahneman, 1973; Tesser and Campbell, 1982). Compte and Postlewaite (2004) are the first to link confidence to economic performance. Economic agents perform better after successes. This paper takes this idea to networks.

I model network formation following Cabrales, Calvó-Armengol, and Zenou (2011).[6] Here, players also choose an interval of types, with whom they would like to interact. They can consequently exclude those whom they do not wish to interact with. Intuitively, agents have a rough understanding on which types they likely encounter on a platform or in a social club. The network formation protocol in this paper thus preserves the intuition of bilateral link formation (Jackson and Wolinsky, 1996). Moreover, some agents may invest more into connections and there is an intensive margin to socialization. Agents distribute their socialization effort equally among different agents across different social clubs. Fershtman and Persitz (2021) provide a micro-foundation for this.

Recent papers stress fragility of social learning (Frick, Ryota, and Ishii, 2020, 2022, 2023). Agents might not learn a true state of the world when they are agnostic about assortativity or have arbitrarily small misperceptions about the type distribution. It is thus crucial to study interactions and socialization in a heterogeneous society, as interactions can fuel the persistence of misperceptions.

The remainder of the paper is organized as follows. Section 2 introduces the model. Section 3 characterizes the equilibrium. Section 4 presents the main result on how societal characteristics shape dispute intensity and total socialization. Section 5 describes various extensions of the model. Section 6 concludes. All proofs are in the appendix.

2 The model

Players: Let $N = \{1, 2, ..., n\}$ be the set of players, where i is the typical player and $n \geq 3$. Player i is of *type* $\theta_i \in [0, 1]$. Let θ_i be drawn from some continuous distribution Φ with probability density function φ. This implies $P(\exists i, j : \theta_i = \theta_j) = 0$ for all $i, j \in N$ and $P(\exists i, j, h \in N : |\theta_i - \theta_j| = |\theta_i - \theta_h|) = 0$. Furthermore, types are immovable. Types 1 and 0 always exist. I refer to them as extremists.[7] If $\theta_i > \theta_j$, then $i > j$. If $\max\{1 - i, i\} < \max\{1 - j, j\}$, I say i is more moderate than j.[8]

Link formation: Each player i nominates an interval of *tolerable types*, $[\underline{t}_i, \bar{t}_i] \in t_i = [0, 1]$. Denote by $\mathbf{t} = (t_1, t_2, ..., t_n)$ vector of tolerance choices. Players feel attachment to

[6] Galeotti and Merlino (2014) use this framework to study investment into job contact networks.

[7] This assumption is not crucial to derive the results presented in this paper, however, it greatly simplifies their exposition.

[8] Note, this concept refers to the type space rather than the distribution. Hence, society can largely consist of extreme types.

their type and suffer from tolerating other types. Let $\tau[(\underline{t}_i - \theta_i)^2 + (\bar{t}_i - \theta_i)^2]$ be the cost of nominating a given range of types $[\underline{t}_i, \bar{t}_i]$ as tolerable, where $\tau > 0$. The parameter τ captures the flexibility of agents.[9]

Player i chooses a socialization effort, $x_i \in X_i = [0, +\infty[$. Socialization entails a constant marginal cost $c > 0$. The socialization profile is a vector $x = (x_1, x_2, ..., x_n)$. Socialization and tolerance generate a weighted network g. I denote the network by an adjacency matrix. Let $K_i(g) = \{j \in N : \theta_j \in [\underline{t}_i, \bar{t}_i]\}$ denote the set of players, whose types i tolerates and denote by $k_i(g)$ the cardinality of this set.[10] Abusing notation, I sometimes write K_i for $K_i(g)$ and k_i for $k_i(g)$ when no confusion arises. Define a weighting function

$$\rho(x, \mathbf{t}) = \begin{cases} \frac{1}{\sum_{j \in K_i} x_j}, \text{ for all } j : \theta_j \in [\underline{t}_i, \bar{t}_i] \text{ and } x_j > 0, \\ 0 \text{ otherwise} \end{cases} \tag{1}$$

A player's socialization effort is divided equally across all agents whom she tolerates. The network g is generated in the following way. Let

$$g_{ij}(x, \mathbf{t}) = \rho(x, \mathbf{t}) x_i x_j \tag{2}$$

Links are symmetric by construction, so $g_{ij} = g_{ji}$ for all $i, j \in N$.[11] Let the total link strength be

$$g_i(x, \mathbf{t}) = \sum_{j \in N} g_{ij}(x, \mathbf{t}) = \rho(x, \mathbf{t}) x_i \sum_{j \in N} x_j \tag{3}$$

The weighted network g captures to whom each agent i is *connected*, i.e., whose types i tolerates and who tolerates her. Moreover, g captures how much players socialize with their connections.

Network definitions: A player i is isolated if $g_{ij} = 0$ for all $j \in N \setminus \{i\}$.

A player's *degree* is the number of her connections, k_i.

A subset of players is a clique $C(g)$, if for all $i \in C(g) \subseteq N$, $g_{ij} > 0$ if $j \in C(g)$ and $g_{ij} = 0$ otherwise. The network g is complete, if all players are in the same clique.

A network g is ordered with respect to types, if for any two nodes i and j, with $i > j$, $\bar{t}_i \geq \bar{t}_j$ and $\underline{t}_i \geq \underline{t}_j$.

A network exhibits strong structural balance if it consists of two cliques and weak structural balance if it consists of more than two cliques.

[9] In Section 5, I discuss the possibility of $\tau_i \neq \tau_j$.

[10] Note, I also use this notation to denote the neighborhood of player i. These coincide straightforwardly in equilibrium, hence the abuse of notation.

[11] Note, the model allows for self-loops.

Benefits: Players do not derive benefits from connections directly. However, connections influence a player's *strength* and *confidence* and increase her benefits from dispute with others. For simplicity, player i is in dispute with j whenever they are not connected, i.e., $g_{ij} = 0$.[12]

I model dispute as a contest between two players. If i is in dispute with j, j is an *opponent* of i. Each player has a perception of how likely it is to win a contest, which depends on the network.

The strength of player i is the sum of the weights of her links, $\lambda_i(g) = \sum_{j \in N} g_{ij} = g_i(x, \mathbf{t})$.[13] An agent's strength comprises of how many connections she has and their weight.[14]

Besides her strength, an agent's connections influence how confident she is to win a contest. I posit that players become overly confident in their chances of winning a dispute when their opponent is also in dispute with their connections. More formally, let $\lambda_{ij}(g) = \sum_{j \in N} \text{sgn}(g_{ih})(1 - \text{sgn}(g_{hj}))$ denote the number of i's neighbors, who are in dispute with j. The function sgn denotes the sign function, which equals one if $g_{ih} > 0$ and zero otherwise.

Let $f(\lambda_i(g), \lambda_j(g), \lambda_{ij}(g))$ denote player i's expected benefits from dispute with j. I omit "expected" when no confusion arises. Call $f(\cdot, \cdot, \cdot)$ the contest success function (henceforth: CSF). For convenience, I write λ_i for $\lambda_i(g)$ and $f(\lambda_i, \lambda_j, \lambda_{ij})$ for $f(\lambda_i(g), \lambda_j(g), \lambda_{ij}(g))$ when no confusion arises. The function $f(\cdot, \cdot, \cdot)$ is strictly increasing in its first argument, the strength of the player, and concave. Moreover, $f(\cdot, \cdot, \cdot)$ is decreasing in its second argument, the strength of the opponent. Hence, stronger opponents lower the expected benefits from dispute. I normalize $f(\lambda_i, \lambda_j, \lambda_{ij}) = 0$ for all $\lambda_i, \lambda_j : \lambda_i = \lambda_j$ and $\lambda_{ij} = 0$.

The parameter λ_{ij} determines how confident players become through their interactions. In particular, players' confidence increases through having mutual opponents with their connections. Formally, let $f(\cdot, \cdot, \cdot)$ be increasing in the third argument. Some parameter β captures the relative weight of the confidence channel.[15] For higher values of β, agents can derive more confidence through mutual opponents with their neighbors. Some parameter $\alpha \in [0, 1)$ captures the perception bias. Similar to Compte and Postlewaite (2004), the perception bias, α, increases an agent's confidence. Note, perceptions need not be correct in this game and opponents can both expect to win the dispute. Interactions distort perceptions. For higher levels of α, each additional neighbor with mutual opponents is more valuable to the

[12] In Section 5, I discuss initiation of dispute.

[13] Note, omitting opponents' friends, whom a player is also an opponent of, is a normalization and does not qualitatively affect the results. Moreover, I can study a version where higher order neighbors also contribute to an agent's strength.

[14] Usually, popularity is a measure of the number, and potentially the weight, of incoming links. Strengths in this model, however, depend on the weight of undirected connections between players.

[15] This can represent agents' use of the availability or representativeness heuristic (Tversky and Kahneman, 1973), since agents tend to interact more with their connections.

agent. Since the game is static, I require confidence to be concave in the number of mutual opponents an agent has, i.e., $\alpha \in [0, 1)$. Since confidence is only one component of the benefit function, I require a "weight" of the confidence channel, β.

Definition 1 *The normalized CSF in ratio form is*

$$f(\lambda_i, \lambda_j, \lambda_{ij}) = \frac{\lambda_i^\phi}{\lambda_i^\phi + \lambda_j^\phi} - \frac{1}{2} + \beta\lambda_{ij}^\alpha \tag{4}$$

and in difference form

$$f(\lambda_i, \lambda_j, \lambda_{ij}) = \frac{1}{1 + e^{\phi(\lambda_j - \lambda_i)}} - \frac{1}{2} + \beta\lambda_{ij}^\alpha \tag{5}$$

The parameters ϕ, β and α parameterize the CSF. Higher values of $\phi \in [0, 1]$ favor stronger agents.

For the analysis, it is convenient to define a notion of how efficient dispute technology is subject to the relative importance of confidence, β, and the perception bias, α. Define $\delta(y) = f(\lambda_i, \lambda_j, y) - f(\lambda_i, \lambda_j, y-1)$ for all $y \in \{1, ..., n-2\}$.[16] The function $\delta(\lambda_{ij})$ is therefore the efficiency of coordination for a given λ_{ij}. I say agents *gain confidence through connections.* Define $\underline{\delta} = \min_{y \leq n-2} \delta(y)$. The parameter $\underline{\delta}$ thus captures the lowest coordination gains.[17]

Let $s_i \in S_i = S \equiv t \times X$ denote the strategy of player i. Strategies comprise of the socialization effort and the tolerance choice. Denote by $s = (s_i, s_{-i})$ the strategy profile, where s_{-i} denotes the strategies of all players, other than i.

The utility of player i is given by

$$u_i(s) = \sum_{j \notin K_i(g)} f(\lambda_i(g), \lambda_j(g), \lambda_{ij}(g)) - cx_i - \tau\left[(\theta_i - \underline{t}_i)^2 + (\theta_i - \bar{t}_i)^2\right] \tag{6}$$

A strategy profile s^* is a *Nash equilibrium*, if

$$u_i(s_i^*, s_{-i}^*) \geq u_i(s_i^*, s_{-i}') \; \forall i \in N \text{ and } s' \in S \tag{7}$$

A strategy profile s^* constitutes an *interior equilibrium*, if there exist i and j, such that $g_{ij}^* > 0$.

Total socialization is the sum of individual socialization efforts, $\sum_{i \in N} x_i$.

Dispute intensity is given by $\iota(s) = \sum_{i \in N} \sum_{j \neq i} \lambda_i(1 - \text{sgn}(g_{ij}))\lambda_j$. In this context, dispute intensity is a measure of polarization as in Esteban and Ray (1994). Dispute intensity

[16] Note, $n-2$ is the maximum number of agents, an individual can have a mutual opponent with.

[17] Given the assumptions, $\delta(\cdot)$ is monotonically increasing in it's argument and concave. Hence, $\underline{\delta} = \delta(n-2)$.

increases on the extensive margin when there are more disputes in society. Dispute intensity increases on the intensive margin if players' strengths increase.

Define a *dense society* such that $n \to \infty$ and $P(\exists i \in N : \theta_i \in [\theta - \epsilon, \theta + \epsilon]) \to 1$.

Foundations: Here, I discuss the foundations of the modelling assumptions.

Types: In my model, types represent a player's views of the world, their beliefs or convictions. Those views are typically immovable in the short run. For instance, individuals take a vaccine before a wave of infections.

Tolerable types: Individuals often have some understanding of others' views through their social media profile or the social club they are in. To some extent, individuals can judge other's views and can decide whom to interact with.

Socialization efforts: Individuals dedicate part of their time to interacting with others. In this context, socialization efforts represent the time spent on social media. If an individual spends more time on social media, she interacts more frequently with other users.

Tolerance: Players feel attachment to their type and suffer from tolerating different views. In this sense, agents have identity based payoffs and can modify their identity at a cost (Akerlof and Kranton, 2000).

Strength: An agent's strength is a measure of how accepted her views are. Connections only occur between players who tolerate each other and support each others' views.

Confidence: Agents often overestimate their success chances or the information available to them (Zacharakis and Shepherd, 2001; Busenitz and Barney, 1997). This extends to friends (Tesser and Campbell, 1982). Agents might mistakenly view their connections as representative of the population or they are more salient (Tversky and Kahneman, 1973).

Dispute as a contest: Dispute is a way to restore one's challenged identity (Akerlof and Kranton, 2000). Alternatively, disputes are attempts to gain control over discussions. Naturally, more confident agents, or agents with more support, believe to likely "win" a dispute.

3 Equilibrium characterization

In this section, I characterize the equilibrium of the game. I present when any equilibrium network exhibits (weak) structural balance, or the neighborhoods of players in dispute overlap. First, note existence of a Nash equilibrium.

Proposition 1 *A Nash equilibrium of the game always exists. Moreover, if an interior Nash equilibrium exists, it is unique.*

The empty network trivially constitutes an equilibrium of the game. No player gains from tolerating others when no other player tolerates her. To show existence of an *interior equilibrium*, i.e., an equilibrium where some agents are connected, it is sufficient to construct a Nash equilibrium for arbitrary parameters. Note, players only tolerate those, who tolerate them as well. Otherwise, one player can tolerate a smaller interval of types and obtain a higher utility. One can thus construct an equilibrium by letting an extremist choose her interval of tolerable types and repeating the process for the all remaining players. By construction, there is no profitable deviation. Since the linking decision and the socialization decision are intertwined, there exists a unique optimal socialization vector for any network g. An interior equilibrium of the game must therefore exist, whenever tolerance is not too costly.

Next, I address uniqueness of the interior equilibrium. In the model, endogenous tolerance decisions determine the unique socialization profile. Showing uniqueness of an equilibrium thus reduces to showing uniqueness of the profile of tolerance intervals. If there were multiple interior equilibria, at least one player is indifferent between tolerating some type and not tolerating her, or she is indifferent between tolerating some type and tolerating another type. The benefits of a connection must consequently equal exactly the additional cost of tolerance, or establishing either of two connections yields exactly the same net benefits. However, agents always differ somewhat in their views, i.e., their type. This condition is thus never met and an interior equilibrium must be unique.

Having established existence of an equilibrium and uniqueness of the interior equilibrium, I next provide a full characterization of the equilibrium network. A crucial component of the model is how confidence, which players derive through their connections, shapes the network and thus economic outcomes. As players derive more confidence through having mutual opponents with their connections, players want to choose more similar neighborhoods to those of their connections. For a higher weight of the confidence channel β, or the perception bias α, agents gain more from coordinating their neighborhoods, i.e. $\delta(y)$ is higher for any possible value of y. An increase in β, or α, thus implies an increase in $\underline{\delta}$. For higher α or β, I say agents gain more confidence through connections.[18]

In the next Proposition, I provide a full characterization of the Nash equilibrium.

Proposition 2 *Any Nash equilibrium network is ordered. Moreover, there exist thresholds* δ^* *and* δ^{**}*, with* $\delta^{**} \geq \delta^*$*, such that*

(i) if the lowest coordination gains are sufficiently high ($\underline{\delta} \geq \delta^{**}$*), the interior Nash equilibrium network exhibits strong structural balance;*

[18]Note, when agents do not gain confidence through connections, i.e., $\beta = 0$, $\delta(y) = 0$ for all $y = \{1, 2, ..., n-2\}$. This case is equivalent to the standard rent seeking model in Tullock (1980).

(ii) if the lowest coordination gains are high, but not too high, ($\delta^{**} > \underline{\delta} \geq \delta^*$), any Nash equilibrium network exhibits weak structural balance;

(iii) otherwise, players' neighborhoods overlap.

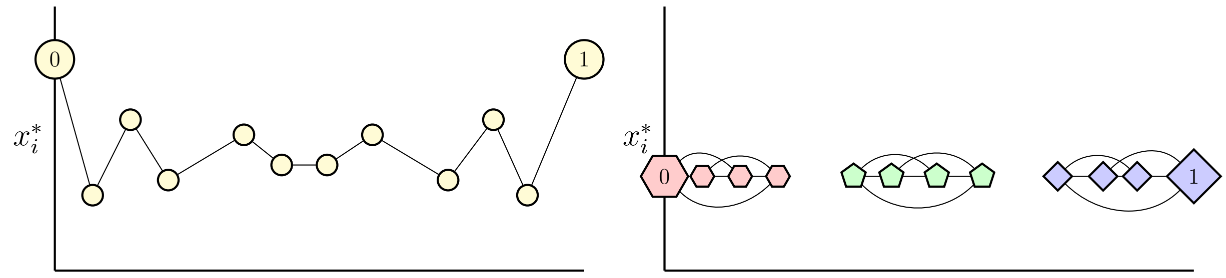
Proposition 2 provides a sharp characterization of the equilibrium network. A first crucial result is that any equilibrium network is ordered. Since tolerance is costly, agents prefer to interact with closer types. They consequently choose connections who are as similar as possible subject to the confidence they can gain. The network is ordered with respect to types.

Next, I establish conditions for when any equilibrium network exhibits strong structural balance. If incentives to coordinate are sufficiently strong, agents are willing to compromise a lot for connections with mutual opponents to reap the benefits from confidence. Moreover, the complete network cannot be an equilibrium, since benefits stem solely from disputes. Hence, there must exist some threshold, δ^{**}, above which the network comprises of exactly two cliques who are in dispute with each other. Each player enjoys the additional benefits from confidence through each connection in her clique more than the additional tolerance cost those connections require.

The intuition behind the statement (ii) of Proposition 2 is similar. As connections with mutual opponents increase agents' confidence more, the returns from forming cliques outweigh the higher cost of tolerating players more dissimilar players in the clique. For smaller coordination gains, $\underline{\delta}$, agents do not tolerate as much and more than two cliques emerge. The equilibrium network exhibits weak structural balance.

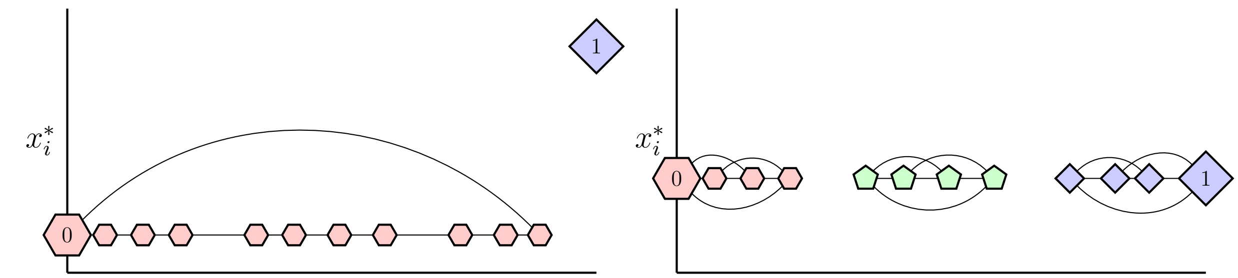
Statement (iii) of Proposition 2 considers the case when agents cannot derive too much confidence through their connections. As established previously, the equilibrium network is ordered. While players may still benefit from having connections with mutual opponents, the gains from confidence of establishing connections are relatively small in comparison to the additional tolerance costs they require. Hence, some players prefer to sponsor cheaper connections to closer agents and there are overlaps in the neighborhoods of players. The characteristics of society determine the value of $\underline{\delta}$. Therefore, the result holds generally. Figures 1 and 2 illustrate examples.

Proposition 2 establishes which network emerges in equilibrium in a given society. Moreover, it highlights conditions for players to form cliques in equilibrium. A natural question is how the network influences economic outcomes. Confidence and socialization costs determine the minimum coordination gains in the economy. The next section links confidence and socialization costs to dispute intensity.



(a) $\beta = 0$ (b) $\beta = 1$

Figure 1: $f(\cdot,\cdot,\cdot) = \frac{\lambda_i^\phi}{\lambda_i^\phi + \lambda_j^\phi} - \frac{1}{2} + \beta\lambda_{ij}^\alpha$, $c = 1$, $\tau = 1$



(a) $\beta = 8$ (b) $\beta = 1$

Figure 2: $f(\cdot,\cdot,\cdot) = \frac{\lambda_i^\phi}{\lambda_i^\phi + \lambda_j^\phi} - \frac{1}{2} + \beta\lambda_{ij}^\alpha$, $c = 1$, $\tau = 1$

4 Confidence, technology, and dispute intensity

This section addresses the comparative statics on confidence and technology. By Proposition 1, there exists a unique interior equilibrium. I study the interior equilibrium, since comparative statics one the empty network are trivial. Abusing notation, I write s^* for the interior equilibrium whenever it exists.

Dispute intensity depends on the number of disputes in society, the extensive margin, as well as the strength of players in dispute, the intensive margin. Socialization efforts consequently translate to higher dispute intensity on the intensive margin. I first establish a general result on socialization efforts.

Lemma 1 *In equilibrium, if i has more neighbors than j, $k_i^* < k_j^*$, then i chooses a lower socialization effort than j, $x_i^* < x_j^*$.*

Lemma 1 links a player's degree to her socialization effort. Players of higher degree exert lower socialization efforts. The reason is twofold. First, those players are in fewer disputes. Their strengths increase their benefits from disputes in fewer instances, thereby reducing the returns to their socialization efforts. Second, a player's strength increases in the socialization efforts of her neighbors. In a sense, players of higher degree enjoy more spillovers through their neighbors and need not socialize as much themselves. It is thus optimal to choose a lower socialization effort.

A critical parameter in the model is how how much confidence players derive through their connections. How confident agents become through their neighbors shapes their tolerance choice and consequently the number of disputes in equilibrium. The effect on dispute intensity depends on the change in the number of disputes and how this change influences the socialization efforts. The next proposition establishes the comparative statics on the confidence channel.

Proposition 3 *As agents gain more confidence through connections (β or α increase),*

(i) if the network consists of cliques ($\delta^* > \underline{\delta}$), dispute intensity decreases;

(ii) otherwise, dispute intensity increases.

Note first, an increase in the benefits from confidence implies greater incentives to coordinate on mutual opponents. This implies players choose smaller tolerance intervals on average. By Lemma 1, total socialization, the intensive margin of dispute intensity, and the number of disputes, the extensive margin of dispute intensity, go in the same direction. The effect on dispute intensity thus depends on the change in the number of disputes in society.

First, consider the case where the intrinsic network consists of cliques. As agents derive more benefits through confidence, incentives to coordinate on mutual opponents increases. Players are willing to tolerate more in order to reap the greater benefits from confidence. Since the intrinsic network consists of cliques, at least one clique grows larger. There are consequently fewer disputes in society and dispute intensity decreases on the extensive margin. By Lemma 1, agents with more connections exert lower socialization efforts. Agents in the large clique reduce their socialization efforts accordingly. The benefits from dispute are increasing in an agent's strength, however, less so for stronger agents. Agents who are in more disputes consequently increase their socialization efforts by less than others decrease their efforts. Dispute intensity decreases on the intensive margin as well. Statement (i) of Proposition 3 follows.

Next, consider the case of an overlapping society. Again, increasing the benefits from confidence incentivizes coordinating on mutual opponents. Some agents consequently tolerate more in one direction. Since the total cost of tolerance is convexly increasing in the distance to the own type, agents choose a smaller tolerance interval. In a sense, agents tolerate fewer types in order to establish connections to more distant types with many mutual opponents.

The overlaps in neighborhoods of players in dispute consequently narrow. Society moves towards a network consisting of distinct cliques. The number of disputes in society increases, thereby increasing dispute intensity on the extensive margin. By Lemma 1, the intensive margin pushes towards higher dispute intensity as well. Players are in more disputes and thus choose higher socialization efforts on average. Dispute intensity thus decreases on the intensive margin and the statement (ii) follows.

The emergence of social media platforms arguably decreased barriers to interactions, i.e., reduced socialization costs. It is thus natural to study how changes in the socialization cost influence economic outcomes. Unlike the case of confidence and tolerance, socialization costs shape the network through two possible channels. First, higher socialization costs crowd out socialization. Second, socialization costs determine how beneficial it is to coordinate on mutual opponents. Players may thus adapt their tolerance intervals to changes in the socialization cost. Proposition 4 addresses the effects on dispute intensity and total socialization.

Proposition 4 *If socialization costs increase, total socialization decreases. Suppose agents gain confidence through connections ($\beta > 0$). Then,*

(i) dispute intensity decreases for low and high socialization costs;

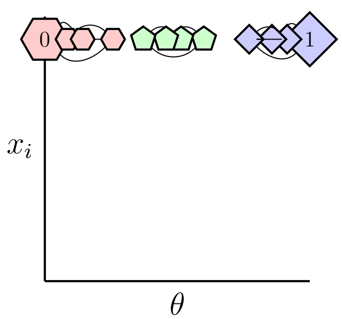
(ii) dispute intensity increases in a dense society if, and only if, overlaps in neighborhoods are small ($\delta^* > \underline{\delta} > \tilde{\delta}$).

First, an increase in the socialization cost always crowds out socialization efforts. Note, socialization efforts may increase when agents are in many more disputes. However, since an increase in the socialization cost alters the incentives for coordination, some players are in fewer disputes whenever others are in more disputes. Returns to socialization diminish and total socialization necessarily decreases.

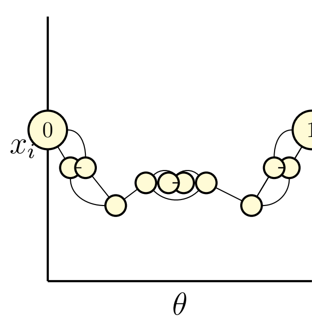
The effects on dispute intensity are *ex ante* ambiguous. On the one hand, higher socialization costs crowd out socialization and dampen dispute intensity on the intensive margin. On the other hand, changes in the socialization costs potentially alter the tolerance intervals of agents. In particular, if socialization costs are low, all players exert high socialization efforts and are strong. Since agents are relatively similar in their strengths, players derive little benefits through their strengths. The confidence channel, however, remains unaffected and agents coordinate their neighborhoods to reap the benefits from confidence. An increase in the socialization cost then reduces the incentives for coordination and crowds out socialization. Consequently, dispute intensity decreases on the extensive, as well as the intensive margin.

If linking costs are high, all agents are of relatively low (and thus relatively similar) strength. Again, returns from investing in the own strength are low and agents coordinate more to reap benefits from confidence. The network consists of cliques. An increase in the socialization cost increases the coordination incentive further and at least one clique becomes larger. Thus, there are fewer disputes in society and dispute intensity decreases on the extensive margin. Higher socialization costs crowd out players' socialization efforts and thus their strengths. Dispute intensity decreases on the intensive margin. The statement follows.

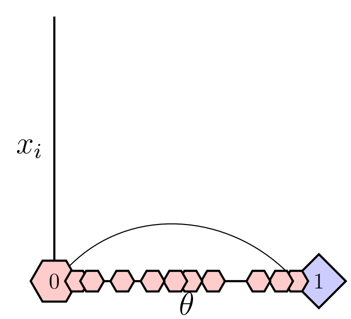
For moderate socialization costs, the effects on the extensive and intensive margin potentially go in opposite directions. While higher socialization costs always crowd out total socialization, it may increase the number of disputes in society. To abstract from special realizations of types, this result relies on a dense society, i.e., all areas on the type space are populated. As overlaps in neighborhoods become smaller, the number of disputes in society increases. Hence, dispute intensity increases on the extensive margin, yet, decreases on the intensive margin as socialization costs crowd out players' strengths. For sufficiently small overlaps in neighborhoods, an increase in the socialization costs results in many more disputes, while agents reduce their socialization effort only by little. The effect of more disputes thus outweighs reduced socialization and dispute intensity decreases. The statement follows. Figure 3 illustrates an example.



(a) Low c



(b) Moderate c



(c) High c

Figure 3: $f(\cdot,\cdot,\cdot) = \frac{\lambda_i^\phi}{\lambda_i^\phi + \lambda_j^\phi} - \frac{1}{2} + \beta\lambda_{ij}^\alpha$, $c = 1$, $\tau = 1$

The analysis thus far uncovers how societal characteristics shape dispute intensity and total socialization. Consequently, it provides some straightforward policy implications. In particular, the equilibrium strategy profile is informative about the effects of interventions. For instance, when the network consists of cliques, increasing socialization costs (or how much confidence players gain through their connections) dampens dispute intensity, while the reverse is true if there are small overlaps in neighborhoods.

Naturally, heterogeneity in types implies differences in how changes in societal characteristics affect the strategies of different types. In this context, it is natural to distinguish between extreme types and moderate types. The next proposition addresses this.

Proposition 5 *In an overlapping society ($\delta^* > \underline{\delta}$), if coordination gains ($\underline{\delta}$) increase, extreme types socialize less and are in fewer disputes.*

Proposition 5 establishes the role of extremists. Coordination gains depend on the characteristics of society as established in Propositions 3 and 4. They emerge either from an increase in confidence (α and β increase), or technological changes (c increases or decreases).

When there are overlaps in players' neighborhoods and coordination gains increase, extreme types become more desirable connections. This is because those types are in dispute with many others. More moderate types can consequently reap larger benefits from having mutual opponents with them and are willing to compromise more for extreme types. In a sense, extreme types drag more moderate types into an extreme group. Those moderate types establish fewer connections and exert higher socialization efforts (Lemma 1). Extreme types, however, establish more connections and thus need not socialize as much. Moreover, they are in fewer disputes and the statement follows.

This result is somewhat similar to the case of intolerant extremists in Genicot (2022). When extremists are less tolerant, moderate types compromise for extremists and consequently choose extreme actions. Polarization arises. Here, players pay a cost for tolerating others. As having mutual opponents becomes more profitable, the benefits from linking to extremists increase. Extremists thus drag moderates into more extreme groups and polarization arises.

A direct implication of Proposition 5 is that the effects on extreme types' strategies go in the opposite direction as the effects on societal outcomes. In particular, as dispute intensity increases, extreme types socialize less and are in fewer disputes. In this sense, extreme types dampen dispute intensity. The reverse is true for decreases in dispute intensity. Interestingly, extremists drag more moderate types in an extreme clique, but do so through their type rather than their action. Influencing extremists' behavior might thus be ill advised and can lead to adverse effects on societal outcomes.

5 Discussion

In this section, I discuss several extensions of the model and show that the results remain qualitatively similar. First, I introduce some additional concepts.

There is a path in g from i to j, $p_{ij} = 1$, if either $g_{ij} > 0$, or there are m players $j_1, ..., j_m$, distinct from i and j, with $g_{ij_1} > 0$, $g_{j_1 j_2} > 0, \cdots, g_{j_m j} > 0$. The length of the path, $l(p_{ij})$, is one in the former case and $m + 1$ in the latter case. Denote the weight of the path by $w_{ij} = g_{ij_1} g_{j_1 j_2} \cdots g_{j_{m-1} j}$. If there exists no path from i to j, $p_{ij} = 0$.

The network is connected, if one path encompasses all players.

Alternative strengths: One can think of an *adjusted Contest Success Function*, where common connections of two players in dispute do not influence their strengths. In particular, consider $\mu_i = \lambda_i - \sum_{h \in N} g_{ih} \operatorname{sgn}(g_{hj})$. One can use μ_i instead of λ_i in the CSF to obtain similar results. Note also, for the case of $\underline{\delta} > \delta^*$, $\mu_i = \lambda_i$. Alternatively, consider a model where higher order neighbors contribute to the strength of players. In particular, denote by $P_i^m = \{j \in N : \exists p_{ij}, \text{ with } l(p_{ij}) \leq m\}$ the set of players, to whom a path from i exists of length m or less. Let $w_{ij} = w_{ii_1} w_{i_1 i_2} \cdots w_{j_m j}$ denote the weight of path p_{ij} of length $l(p_{ij}) = m$. Define $\mu_i = \sum_{j \in P_i^m} w_{ij}$. One can simply use μ_i instead of λ_i in the CSF.

Initiation of dispute: Suppose agents pay a finite cost for initiating dispute, D. Then, $g_{ij} = -1$ indicates that i initiates dispute with j and $\bar{g}_{ij} = -1$ if $\min\{g_{ij}, g_{ji}\} = -1$. A value $g_{ij} = 0$ denotes a neutral relationship between i and j. Moreover, let $\lambda_i(\bar{g}) = \sum_{h \in K_i(\bar{g})} g_{ih}$ be the strength of player i, i.e., the sum of positive link weights. I posit that players expect to win a prize, V. We can formulate the following result.

Proposition 6 *If agents gain confidence through connections, there exists a threshold $\bar{V}$, such that for a prize $V \geq \bar{V}$, each agent i is in dispute with everyone outside her tolerable type window, $\theta_j \notin [\underline{t}_i, \bar{t}_i]$.*

In the model with costly dispute initiation, agents will initiate dispute, once the expected benefits from dispute exceed its initiation cost D. If i is in dispute with j, either i expects positive benefits from dispute, h expects positive benefits from dispute, or both. Hence, if the prize V is large enough, either i initiates dispute with h, or h initiates dispute with i and the proposition follows. Note, if I impose initiating disputes as a tie breaker, agents need not gain confidence through their neighbors to establish Proposition 6.

Another variable of interest is how economic outcomes depend on how easily agents can tolerate others. This shapes outcomes through the number of connections an agent establishes in equilibrium. The next proposition summarizes the result.

Proposition 7 *Dispute intensity is decreasing in the flexibility of agents (τ).*

In this case, agents suffer less from modifying their identities. Agents tolerate a larger interval of types. By Lemma 1, more connections imply lower socialization efforts. Total socialization decreases and dispute intensity thus decreases on the intensive margin. For a similar reason, players are in fewer disputes. Dispute intensity decreases on the extensive margin.

Heterogeneous flexibility: The baseline model assumes homogeneity in agents' flexibility. In principle, allowing arbitrary flexibility of agents would result in more connections for more flexible agents. A natural way to introduce heterogeneity to the model is assuming more extreme types to be more *stubborn* or *flexible*. On the one hand, extremists may be more stubborn and thus be less willing to tolerate others' types. On the other hand, extremists might be more flexible since they can only compromise in one direction. Then, tolerating other views comes at a lower cost. Genicot (2022) uses a similar approach to impose structure on agents' willingness to compromise for others. In particular, let $\tau_i = \underline{\tau} + |\theta_i - \frac{1}{2}|$, i.e., more extreme players pay a higher cost for compromise or $\tau_i = \max\{\bar{\tau} - |\theta_i - \frac{1}{2}|, 0\}$. Since links are formed bilaterally, any equilibrium network is ordered if there is heterogeneity in agents' flexibility and $\underline{\tau}$ is sufficiently high. The model with heterogeneous flexibility thus produces qualitatively similar results to the baseline model. Moreover, heterogeneity pushes towards cliques. This is because either extremists only tolerate very extreme types or only very extreme types tolerate them.

General type space: Let $\theta_i \in [\underline{\theta}, \bar{\theta}]$.One can directly see that restricting the analysis to an arbitrary bounded type space does not yield qualitatively different results.

Repeated interaction: The model abstracts from the possibility that agents update their intrinsic types. In most contexts of social media outlets, societies are large and agents cannot possibly know all users. Moreover, new agents are born over time. Here, I show formally that the results hold when sufficiently many new agents enter society each period. Suppose time moves discretely in an infinite horizon. Moreover, previous dispute resolves. Hence, if in period σ, i and j are in dispute, then $\lambda_{ij} = \lambda_{ji} = 0$ in period $\sigma + 1$. One can establish the following result.

Proposition 8 *There exists $\bar{m}$ such that if more than $\bar{m}$ players are born each period, no player alters her interval of tolerable types.*

The intuition behind Proposition 8 is simple. New players are born into others' tolerance interval and establish connections with those players. New players, once more, boost players' confidence. Thus, there must exist some threshold of new agents, beyond which no agent adapts her tolerance interval.

Learning: Suppose time moves discretely and the same game is played each period. Moreover, suppose agents update their intrinsic types in a DeGroot (1974) manner. In particular, let θ_i^σ denote i's type and $\bar{\theta}_i^\sigma = \sum_{j\in K_i(g^\sigma)} \theta_j^\sigma / k_i(g^\sigma)$ the average type of her connections at time σ. Then, $\theta_i^{\sigma+1} = \gamma\theta_i^\sigma + (1-\gamma)\bar{\theta}_i^\sigma$, where $1 \geq \gamma > 0$. Agents match the types of their connections, however, types are sticky. I formulate the following result.

Proposition 9 *If $\underline{\delta} > \delta^*$ in the first period, in the long run, each agent's type converges to the average type of her clique.*

Clearly, the network consists of cliques, since $\underline{\delta} > \delta^*$. When agents update their intrinsic types, they become more similar to everyone they are connected with and more dissimilar to everyone else. Hence, no player has an incentive to tolerate some other player in the second period, and so on. Since agents update their type each period and neighborhoods remain identical, their type converges to the average types of their connections. If there are overlaps in neighborhoods, converging to a consensus is possible as shown in Bolletta and Pin (2020).

Geography of conflict: Spatial proximity between countries is often associated with the likelihood that those countries are in an alliance. Consider the following variation of the model. Let agents be located on a unit square. In particular, let $\theta_i = (\underline{\theta}_i, \bar{\theta}^i)$, where $\underline{\theta}_i \in [0,1]$ and $\bar{\theta}^i \in [0,1]$ denote the horizontal and vertical location of agent i respectively. As in the baseline model, θ_i and θ^i are drawn from some continuous distribution and may correlate. Fix four extremists at $(0,0)$, $(0,1)$, $(1,0)$ and $(1,1)$ and suppose agents choose a tolerance range around their ideal point in the unit square, t_i.[19] Hence, we can disregard the confidence channel entirely and the utility of agent i is given by

$$u_i(s) = \sum_{j \notin K_i(g)} f(\lambda_i, \lambda_j) - cx_i - \tau t_i^2 \tag{8}$$

One can establish the following result.

Proposition 10 *An equilibrium of the game always exists and if an interior equilibrium of the game exists, it is unique.*

The empty network exists trivially. If each agent chooses a tolerance range of zero, no player has a profitable deviation. Since I draw coordinates from continuous distributions, each node is equally distant to two other nodes with probability zero. One alliance is thus always easier to establish and the equilibrium is unique.

There are several important ways in which the two-dimensional model differs from the baseline model. Some agents potentially tolerate types, who do not tolerate them in equilibrium. This is because agents can no longer compromise in one direction and tolerate types

[19] Allowing agents to compromise more in one direction would make the model insolvable, since agents could compromise in arbitrarily many directions.

around the own type. For a similar reason, introducing confidence no longer alters agent's tolerance decisions and we can abstract from confidence in the two-dimensional model.

However, some results from the one-dimensional model generalize to the two-dimensional case. First, there is conflict in any equilibrium. Moreover, an agent's ability to form alliances depends on her location and the location of other agents. Hence, some agents might form alliances with relatively distant types while others only need to compromise little to establish connections. This might push agents towards alliances with more extreme types.

6 Conclusion

I study a game where heterogeneous players form connections and derive benefits from dispute with others. Heterogeneity stems from differences in agents' views. Tolerating more distant views comes at an increasingly larger cost. An agent's socialization effort determines the strengths of her connections to types whom she deems tolerable, and who tolerate her. An agent's strength and confidence determine her benefits from disputes, which are contests between two players. Agents grow confident in winning disputes when their connections are also in dispute with their opponents. In this sense, connections feed an endogenous echo chamber effect in the model. Any Nash equilibrium network is ordered with respect to types. Moreover, the network consists of cliques when the confidence channel is sufficiently strong. Otherwise, agents' neighborhoods overlap.

I show how societal characteristics shape economic outcomes through interactions. Dispute intensity is non-monotonic in the confidence channel and the socialization costs. If the network consists of echo chambers, more confidence dampens dispute intensity. Otherwise, dispute intensity increases in how much confidence agents gain through connections. Higher socialization costs lead to higher dispute intensity when overlaps in neighborhoods are small and dampen dispute intensity otherwise. The equilibrium strategy profile is thus informative about the optimal policy intervention. Interestingly, extremists' behavior dampens the effect of interventions on societal outcomes.

A stark implications of my findings is that confrontational forms of dispute, which have become more common on new social media platforms, contribute substantially to the rising polarization in society. Indeed, easier access to social media can result in higher dispute intensity. Social media platforms and policy makers alike may thus want to decrease hurdles to access only when society is on the verge of echo chambers.

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Appendix

Proof of Proposition 1. To establish existence, it is sufficient to construct an equilibrium with an arbitrary configuration of parameters. Fix some θ, ϕ, τ, α and β. Take agent 1 and fix $[\underline{t}_1, 1]$. In equilibrium, if $i \notin [\underline{t}_1, 1]$, then $\bar{t}_i < 1$, since $g_{i1} = 0$ in any case and $\tau > 0$. Next, take $i > j$ for all $j \in N$ with $\theta_j < \underline{t}_1$ and let her choose $[\underline{t}_i, \bar{t}_i]$. By construction, no profitable deviation emerges for 1, since $g_{i1} = 0$ in any case. For each $i' > i$, $\underline{t}_{i'} > \underline{t}_i$, since $\tau > 0$. Let each i' choose $[\underline{t}_{i'}, \bar{t}_{i'}]$. By construction, no profitable deviation emerges. If $\underline{t}_i = 0$, let each $i \in N$ choose x_i accordingly. From our assumptions on $f(\cdot,\cdot,\cdot)$, for any given network, there is a unique profile of optimal socialization efforts, x. Hence, this constitutes an equilibrium s^*. Otherwise, take $j < \underline{t}_i$, where $j \geq h$ for all h, with $\theta_h < \underline{t}_i$ and repeat the process until, for some m, $\underline{t}_m = 0$. Let each $i \in N$ choose x_i accordingly. The resulting network constitutes an equilibrium, which proves the first statement of the proposition.

Next, I address uniqueness of the interior equilibrium. First, note that for any configuration of $\mathbf{t} = t_1 \times ... \times t_n$, there exists a unique optimal socialization vector x. Thus, the equilibrium is unique, if there exists a unique $\mathbf{t}^*$. Suppose *ad absurdum* there exist s^* and $s^{*'}$ which constitute an equilibrium of the game. Hence, there must exist at least one player, say i, who is indifferent between tolerating j or h, or who is indifferent between tolerating j and not tolerating j. I address the cases separately.

Case 1: Suppose i is indifferent between tolerating j and not tolerating j, where $j > i$ wlog. Denote by h the highest type whom i tolerates in case she does not tolerate j. Denote g^* the equilibrium network where $j \in K_i(g^*)$ and $g^{*'}$ the equilibrium where $j \notin K_i(g^{*'})$. Agent i is indifferent between tolerating j and not tolerating j if $\sum_{l \notin K_i(g^*)} f(\lambda_i^*, \lambda_l^*, \lambda_{il}^*) - \sum_{l \notin K_i(g^{*'})} f(\lambda_i^{*'}, \lambda_l^{*'}, \lambda_{il}^{*'}) = \tau[(\theta_j - \theta_i)^2 - (\theta_h - \theta_i)^2]$. By the assumptions on the distribution of types, this cannot hold for any $i, j \in N$. Hence, no player i is indifferent between tolerating and not tolerating another agent j.

Case 2: i is indifferent between tolerating j and tolerating h, where $h > i > j$ wlog. By assumption, $P(\exists i, j, h \in N : \{|\theta_i - \theta_j| = |\theta_i - \theta_h|\}) \to 0$. Hence, either j or h, say h, must have more mutual opponents with i. Suppose $g_{ij}^* > 0$ and $g_{ih}^* = 0$, while $g_{ij}^{*'} = 0$ and $g_{ih}^{*'} > 0$. This implies $\sum_{l \in K_i^*} f(\lambda_i^*, \lambda_l^*, \lambda_{il}^*) - \sum_{l \in K_i^{*'}} f(\lambda_i^{*'}, \lambda_l^{*'}, \lambda_{il}^{*'}) = \tau[(\theta_i - \theta_h)^2 - (\theta_i - \theta_j)^2]$, i.e., the additional cost of tolerating h instead of j equals the additional benefits of tolerating h instead of j. Since $f(\cdot,\cdot,\cdot)$ is discretely increasing in its third argument and by the assumptions on the distribution of types, this condition cannot hold, a contradiction. The statement follows. ■

Proof of Proposition 2. I prove the proposition in several steps.

Lemma A-1 *Any Nash equilibrium network is ordered.*

Proof. Suppose *ad absurdum* $\theta_i > \theta_j$ and $\bar{t}_i < \bar{t}_j$ wlog. I distinguish two cases.

Case 1: $\underline{t}_i > \underline{t}_j$. For each h, with $\theta_h \in [\underline{t}_j, \bar{t}_j]$, it must hold that $\theta_j \in [\underline{t}_h, \bar{t}_h]$. Otherwise, $g_{ij} = 0$, and narrowing $[\underline{t}_j, \bar{t}_j]$ is a profitable deviation. Hence, there must exist $h > i$, such that $\theta_h \in [\underline{t}_j, \bar{t}_j]$ and $\theta_h \notin [\underline{t}_i, \bar{t}_i]$. Since $h > j$, $\underline{t}_h \leq \theta_j$. Otherwise, $g_{jh}^* = 0$, a contradiction. This implies either $\bar{t}_i < \theta_h$ or $\theta_i > \bar{t}_h$. $\bar{t}_i < \theta_h$ directly yields a contradiction, since choosing

$\bar{t}_i = \bar{t}_j$ is a profitable deviation for i conditional on $\bar{t}_j \geq \theta_h$. If $\bar{t}_i > \theta_h$, it must be profitable for j to reduce $\bar{t}_j$, at least such that $\bar{t}_j = \bar{t}_i$, a contradiction.

Case 2: $\underline{t}_i < \underline{t}_j$. In this case, there exists $h > i$ and $l < j$ such that $\theta_h \in [\underline{t}_i, \bar{t}_i]$ and $\theta_l \in [\underline{t}_j, \bar{t}_j]$, while $\theta_l \notin [\underline{t}_i, \bar{t}_i]$ and $\theta_h \notin [\underline{t}_j, \bar{t}_j]$. Since compromising is costly, i can increase her payoff by choosing $\underline{t}_i = \underline{t}_j$ and $\bar{t}_i = \bar{t}_j$, a contradiction to the initial assumption. The statement follows. ■

Having established Lemma A-1, it is sufficient to show that one can always find values of $\underline{\delta}$ such that players form cliques. Suppose *ad absurdum* no δ^* and δ^{**} exist. Take some $\underline{\delta} = \delta(n-2) - \delta(n-3)$ wlog. Then, for the largest possible clique $C(g^*)$, $|C(g^*)| = n-1$. Suppose, $\underline{t}_1 = \theta_i$ and $\bar{t}_i = 1$, where $i \leq j$ for all $j \in N \setminus \{0\}$. Then, it is sufficient to establish a contradiction for 1, since she chooses the widest tolerance interval. Her cost saving from deleting a link to i from the clique is $\tau[(1-\theta_i)^2 - (1-\theta_j)^2]$, where $\nexists h$, such that $i < h < j$. This expression is finite, since $[0, 1]$ is compact, a contradiction. This proves existence of of δ^* and δ^{**}. Since a network with strong structural balance comprises of fewer cliques, $\delta^{**} \geq \delta^*$ follows directly. The characterization follows trivially, which concludes the proof of Proposition 2. ■

Proof of Lemma 1. Suppose *ad absurdum* $k_i(g^*) > k_j(g^*)$, $x_i^* > x_j^*$. I distinguish two cases.

Case 1: Suppose $\underline{\delta} \geq \delta^*$. Hence, any equilibrium exhibits at least weak structural balance. This implies, if $j \in K_i(g^*)$ and $k_i = m$, then $k_j = m$. Hence, $g_{ij}^* = 0$. Suppose $k_j = m-1$ wlog instead. By construction, j is in dispute with more player than i. Hence, $\sum_{j \notin K_i(g^*)} f'(\lambda_i, \lambda_l, \lambda_{il}) = \sum_{h \notin K_j(g^*)} f'(\lambda_j, \lambda_h, \lambda_{jh}) = c$. This is a contradiction, since $f(\cdot, \cdot, \cdot)$ is strictly concave in its first argument.

Case 2: Suppose $\delta^* > \underline{\delta}$. Agents' neighborhoods thus overlap. For j with $k_j = m-1$, $\sum_{h \notin K_i(g^*)} f'(\lambda_i, \lambda_h, \lambda_{ih}) = \sum_{l \notin K_j(g^*)} f'(\lambda_j, \lambda_l, \lambda_{jl}) = c$ implies $\sum_{h \in K_i(g^*)} x_h^* < \sum_{l \in K_j(g^*)} x_l^*$. Hence, i's connections must have more connections than the connections of player j. Otherwise, $\sum_{p \notin K_h(g^*)} f'(\lambda_h, \lambda_p, \lambda_{hp}) > c$, a contradiction. Recall, any equilibrium network is ordered. This implies either; (i) $\exists j'$, such that $g_{jj'}^* > 0$ is a profitable deviation for j and j', since $\tau[(\theta_i - \theta_h)^2 - (\theta_i - \theta_{h'})^2] \leq \sum_{h \notin K_i(g^*)} f'(\lambda_i, \lambda_h, \lambda_{ih})$, where $\nexists h''$ such that $h > h'' > h'$, implies $\tau[(\theta_j - \theta_l)^2 - (\theta_j - \theta_{l'})^2] \leq \sum_{l \notin K_j(g^*)} f'(\lambda_j, \lambda_l, \lambda_{jl})$, where $\nexists l''$, such that $l < l'' < l'$, or (ii), $\exists i'$, such that $g_{ii'}^* = 0$ is a profitable deviation for i and i', since $\tau[(\theta_j - \theta_l)^2 - (\theta_i - \theta_{l'})^2] \geq \sum_{l \notin K_j(g^*)} f'(\lambda_j, \lambda_l, \lambda_{jl})$, where $\nexists l''$ such that $l > l'' > l'$, implies $\tau[(\theta_i - \theta_h)^2 - (\theta_i - \theta_{h'})^2] \geq \sum_{l \notin K_i(g^*)} f'(\lambda_i, \lambda_h, \lambda_{ih})$, where $\nexists l''$, such that $l < l'' < l'$. This is a contradiction, which proves the statement. Since the linking protocol allows for self-loops, isolated agents invest in their own strength and the statement holds for isolated agents as well. ■

Proof of Proposition 3. I first establish how the benefits from dispute depend on α and β.

Lemma A-2 *Take some $\alpha > \alpha'$ and $\beta > \beta'$. Then, $\delta(y, \alpha, \beta) > \delta(y, \alpha, \beta') > \delta(y, \alpha', \beta')$ and $\delta(y, \alpha, \beta) > \delta(y, \alpha', \beta) > \delta(y, \alpha', \beta')$ for all $y \in \{0, 1, ..., n-2\}$. Moreover, if $\beta = 0$, $\delta(y, \alpha, \beta) = 0$ for all $y \in \{0, 1, ..., n-2\}$.*

Proof. To prove the statement, note that $\frac{\partial f(x,y,z)}{\partial z} > 0$ by assumption. Moreover, $\frac{\partial \lambda^*_{ij}}{\partial \alpha} > 0$ and $\frac{\partial \lambda^*_{ij}}{\partial \beta} > 0$. From the envelope theorem, I can thus infer $\frac{\partial f(x,y,z)}{\partial \alpha} > 0$ and $\frac{\partial f(x,y,z)}{\partial \beta} > 0$. The first part of the statement follows directly. If $\beta = 0$, the statement follows trivially. The lemma follows. ■

Next, I address how dispute intensity and total socialization change in α and β. There are two cases.

Case 1: $\underline{\delta} < \delta^*$. Since incentives to coordinate are higher for higher levels of δ, it follows directly that there are more disputes. By Lemma 1, players with fewer connections socialize more. Since returns to socialization are diminishing and there are more disputes, it follows trivially that total socialization increases.

Case 2: $\underline{\delta} \geq \delta^*$. In this case, society consists of cliques. As $\underline{\delta}$ increases, $|C_{\bar{m}}(g^*)|$ increases, i.e., the cardinality of the largest clique is larger. Since any equilibrium network is ordered, there are fewer disputes in society. Moreover, marginal returns to socialization are diminishing. Hence, total socialization decreases. Then, it follows directly that $\iota(g^*)$ decreases. The statement follows. ■

Proof of Proposition 4. I prove the proposition in a series of lemmata. First, I address total socialization.

Lemma A-3 *Total socialization is decreasing in c.*

Proof. Note, unless an increase in c alters the neighborhood of at least one player, the statement follows trivially. Suppose instead, at least one player alters her tolerance windows due to an increase in the socialization cost c. If she establishes more connections, the statement holds trivially. Hence, the only concern is when she establishes fewer connections. Suppose *ad absurdum* i severs the tie to one of her neighbors, say j, and increases her socialization efforts. Hence, $\sum_{h \in K_i(g^*)} f'(\lambda_i, \lambda_h, \lambda_{ih}) = \sum_{h \in K_i(g^{*\prime})} f'(\lambda'_i, \lambda_h, \lambda_{ih}) + f'(\lambda'_i, \lambda'_j, \lambda_{ij}) =$

c. By Proposition 3, $x_i^{*'} > x_i^*$ and $x_j^{*'} > x_j^*$, however, since $f'(\cdot,\cdot,\cdot)$ is decreasing, $\sum_{h \in K_i(g^*)} f'(\lambda_i, \lambda_h, \lambda_{ih}) - \sum_{h \in K_i(g^{*\prime})} f'(\lambda_i', \lambda_h, \lambda_{ih}) < 0$. Hence, $f'(\lambda_i', \lambda_j', \lambda_{ij}) < 0$, a contradiction to $f(\cdot,\cdot,\cdot)$ being increasing in its first argument. Statement (i) of Proposition 4 follows. ■

The next Lemma establishes a non-monotonic relationship between dispute intensity and the socialization cost.

Lemma A-4 *For sufficiently low or high c, any equilibrium network exhibits at least weak structural balance.*

Proof. To prove the statement, I first show that for $c \to 0$ and $c \to \infty$, g^* exhibits structural balance. Note, in those cases, $\lambda_i = \lambda_j$ for all $i, j \in N$. This stems from the assumptions of the benefit function. Suppose *ad absurdum* that there exist i, j and h, such that $g_{ij}^* > 0$, $g_{jh}^* > 0$, and $g_{ih}^* = 0$, where $i > j > h$ wlog. This implies $\delta(k_i + 1) - \delta(k_i) < \tau[(\theta_i - \theta_h)^2 - (\theta_i - \underline{t}_i^*)^2]$, or $\delta(k_h + 1) - \delta(k_h) < \tau[(\theta_h - \theta_i)^2 - (\theta_h - \underline{t}_h^*)^2]$, or both. Moreover, $\delta(k_j) - \delta(k_j - 1) \geq \tau[(\theta_j - \theta_h)^2 - (\theta_i - \underline{t}^*)^2]$. Hence, there must exist some player l, such that $g_{il}^* = g_{jl}^* = 0$. Otherwise, the connection to i yields no benefits and j has a profitable deviation in reducing $\bar{t}_h^*$. Since any equilibrium network is ordered, either $l < h$ or $l > i$. As all players are of equal strength, either i or j have a profitable deviation in tolerating l instead, or j has a profitable deviation in cutting the link to i. This is because τ is finite and the type space is bounded, a contradiction. This proves the statement. ■

Next, I address dispute intensity. Suppose c is low and the equilibrium network is structurally balanced. An increase in c has thus two effects. First, it crowds out socialization, which by Lemma A-3 decreases dispute intensity on the intensive margin. Moreover, either no existing link is severed or no absent link is added and dispute intensity decreases. Otherwise, players, on average, form more links, since strengths potentially differ and agents need not coordinate as much. Dispute intensity decreases on the extensive margin.

Next, suppose the network exhibits at least weak structural balance and socialization costs are high. Again, an increase in c crowds out socialization, which by Lemma A-3 decreases dispute intensity on the intensive margin. Moreover, either no existing link is severed or no absent link is added and dispute intensity decreases. Otherwise, players, on average, form more links, since agents are more similar in their strengths and thus coordination on mutual opponents is more beneficial.

It remains to be shown that dispute intensity increases for some moderate c. The next lemma addresses this.

Lemma A-5 *If $\delta^* > \underline{\delta} > \tilde{\delta}$, in a dense society, dispute intensity is increasing in c.*

Proof. First, one can show $\tilde{\delta} < \delta^*$. Suppose otherwise. Then, increasing c implies that agents coordinate more. Since $\underline{\delta} > \delta^*$, the network exhibits structural balance. Hence, more coordination implies fewer disputes and an increase in c dampens dispute intensity, a contradiction.
It remains to be shown that there exists some interval $]\tilde{\delta}, \delta^*]$, such that dispute intensity is increasing in c. Take $\underline{\delta} = \delta^*$. Hence, an increase in c implies fewer overlaps in neighborhoods of players. In particular, suppose $c' = c - \epsilon$, where $\epsilon \to 0$. Hence, there are more disputes in society and dispute intensity increases. By Lemma A-3, total socialization must decrease. However, benefits from adding a neighbor with a mutual opponent increase discretely. Hence, the increase in dispute intensity on the extensive margin must outweigh the decrease on the intensive margin. The statement follows. ■
The proposition follows directly from the lemmata. ■

Proof of Proposition 5. To prove the statement, it is sufficient to establish a contradiction for one of the extremists. Suppose *ad absurdum* $x_1^* > x_1^{*'}$, where $\delta^* > \underline{\delta}^* > \underline{\delta}^{*'}$. By Proposition 3, this implies $k_1(g^*) < k_1(g^{*'})$. Denote by i and i' the most moderate neighbor of 1 under g^* and $g^{*'}$ respectively. Clearly, $i > i'$, so the cost of tolerance is lower for i. However, marginal returns to linking to 1 are higher for $\underline{\delta}^*$. Hence, $i > i'$ can never be the most moderate neighbor of 1, a contradiction. The statement follows. ■

Proof of Proposition 6. To prove the proposition, it is sufficient to show that at least one agent expects positive gross benefits from dispute. Since $g_{ij} > 0$ for some i and $j \in N$, $\lambda_i^* > 0$. Moreover, the Nash equilibrium network is ordered. Hence, $\lambda_{ih}^* > 0$, for all i with $K_i(g^*) \neq \emptyset$. Then, either $f(\lambda_i^*, \lambda_h^*, \lambda_{ih}) > D$, or $f(\lambda_h^*, \lambda_i^*, \lambda_{hi}) > D$, or both if V is large relative to D. This implies either i initiates dispute with h, or h initiates dispute with i. The statement follows. ■

Proof of Proposition 7. To prove the proposition, it is sufficient to show $\sum_{i \in N} x_i^*$ is decreasing, while there are fewer disputes in society. It follows trivially that players choose a larger interval of tolerable types when they become more tolerant, i.e., τ decreases. Consequently, there are fewer disputes in society and dispute intensity, all else equal, decreases on the extensive margin. Second, for a given c, α and β, all players tolerate a larger interval of types. Since agents of higher degree socialize less, total socialization decreases on the intensive margin. Thus, dispute intensity must decrease and the statement follows. ■

Proof of Proposition 8. Denote time by $\sigma = \{1, 2, ..., \Sigma\}$. Types are drawn from the same distribution. Hence, there must exist $\bar{m}$ such that if $\bar{m}$ more players are drawn each period, $P(2k_{i\sigma} \leq k_{i\sigma+1}) = 1$ for all σ. Hence, players are in at least as many disputes. It follows that, $\underline{t}_{i\sigma} = \underline{t}_{i\sigma+1}$ and $\bar{t}_{i\sigma} = \bar{t}_{i\sigma+1}$. Note, if $\bar{m}$ is large, society is dense. The statement follows. ■

Proof of Proposition 9. First, suppose *ad absurdum* that $h \notin K_i(g_1^*)$, yet $h \in K_i(g_\sigma^*)$ for some $\sigma > 1$. For $h \in K_i(g_1^*)$, $|\theta_i^\sigma - \theta_h^\sigma| < |\theta_i^1 - \theta_h^1|$ follows directly from updating. Since any equilibrium network is ordered, updating preserves the order of types over time. We can thus establish the argument for i, with $\theta_i^1 > \theta_k^1$ for all $k \in [\underline{t}_i^1, \bar{t}_i^1]$ wlog. Clearly, $|\theta_i^\sigma - \theta_k^\sigma| < |\theta_i^1 - \theta_k^1|$, where $\nexists l$, with $\theta_k < \theta_l < \theta_i$. From updating it follows straightforwardly that $\theta_k^\sigma \leq \theta_k^{\sigma-1}$ and $\theta_i^\sigma \geq \theta_i^{\sigma-1}$. Costs of tolerating k thus increase over time, a contradiction. Since neighborhoods remain constant over time, it follows trivially that types converge to the average type of their clique. The statement follows ■

Proof of Proposition 10. First, note that the empty network is always an equilibrium. Now suppose *ad absurdum* there exist multiple interior equilibria. Agents tolerate everyone within distance t_i^* of their type. Hence, there must exist at least one agent, say i, who tolerates more in one equilibrium than another, i.e., $t_i^* > t_i^{**}$. This implies, for some j, $t_j^* > t_j^{**}$. Suppose the equilibrium play is $\mathbf{t}^*$. By concavity of $f(\cdot,\cdot)$, either i has a profitable deviation in choosing t_i^{**} instead, or j has a profitable deviation in choosing t_j^{**} instead. This contradiction concludes the proof of the statement. ■